

# Seesaw Type I and III at the LHeC

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**ABSTRACT:** We study the potential of testing the seesaw type-I and III models at the Large Hadron-electron Collider (LHeC), an e-p collision mode at the CERN collider. The e-p collision mode provides an excellent place to study lepton number violating processes  $e^-p \rightarrow N_i j + X \rightarrow e^+ W^- j + X$ ,  $e^-p \rightarrow N_i j + X \rightarrow \tau^\pm W^\mp j + X$  and  $e^-p \rightarrow E_i j + X \rightarrow \tau^- Z j + X$  with  $W$  and  $Z$  into hadron jets. Here  $N_{1,2,3}$  and  $E_{1,2,3}$  are heavy Majorana neutrinos and heavy charged leptons, and  $j$  is a hard hadron jet. Although the process  $e^-p \rightarrow N_i j + X \rightarrow e^+ W^- j + X$  is stringently constrained from neutrinoless double-beta decay, there are solutions where this constraint can be satisfied with sizeable production cross section. With the electron energy  $E_e = 140$  GeV and proton energy  $E_p = 7$  TeV, we find that the cross section for the heavy charged lepton  $E$  production can reach a few  $fb$  when the heavy charged lepton mass  $m_E < 600$  GeV. For the heavy neutrino  $N$  production, the cross section can be as large as a few  $fb$  for the mass scale as high as 1 TeV, higher than what can be achieved by the p-p collision mode of LHC with the same related heavy neutrino couplings.

**KEYWORDS:** The Seesaw Models, Heavy Neutrino, Large Hadron-electron Collider(LHeC).

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## 1. Introduction

Various experiments have now established that neutrinos have masses and mixes with each other [1]. The masslessness of the neutrinos in the minimal standard model (SM) implies that one has to go beyond it to account for this observation. Among a number of possibilities that have been proposed, the most popular ones are the seesaw scenarios [2–17] in which new particles are introduced with masses sufficiently large to make the neutrino masses small.

In the so-called seesaw type-I (ST-I) and type-III (ST-III) models [11–17], the heavy particles responsible for giving masses to the light neutrinos are neutral fermions – the heavy neutrinos. The best way to test seesaw models is to produce the heavy neutrino  $N$ , as well as their charged partners in the case of the ST-III model. Several studies of seesaw models at the LHC in p-p collision mode have been carried out [18–44]. It has been shown that at the LHC single production of heavy seesaw particle with mass of order a few hundred GeV can be probed [38]. For the ST-III model, pair production of heavy seesaw particle up to 1 TeV can be achieved [38–42].

In this work we study the possibility of testing the ST-I and ST-III models at the Large Hadron-electron Collider (LHeC), an possible operation mode with e-p collision for the LHC. The e-p collision mode provides an excellent place to study lepton number violating processes  $e^-p \rightarrow N_i j + X \rightarrow e^+ W^- j + X$ ,  $e^-p \rightarrow N_i j + X \rightarrow \tau^\pm W^\mp j + X$  and  $e^-p \rightarrow E_i j + X \rightarrow \tau^- Z j + X$  with  $W$  and  $Z$  into hadron jets. Although there is stringent constraint from neutrinoless double-beta decay, it only constrains the  $e^-p \rightarrow N_i j + X \rightarrow e^+ W^- j + X$  process. Even for the  $e^-p \rightarrow N_i j + X \rightarrow e^+ W^- j + X$  process, there are solutions with

non-degenerate heavy neutrinos which can satisfy the constraint from neutrinoless double-beta decay. Then the cross sections for the processes  $e^-p \rightarrow N_{ij} + X \rightarrow e^+W^+j + X$  ( $i = 1, 2, 3$ ) can be sizeable. With the electron energy  $E_e = 140$  GeV and proton energy  $E_p = 7$  TeV, we find that the cross section for the heavy charged lepton  $E$  production can reach a few  $fb$  when the heavy charged lepton mass  $m_E < 600$  GeV. For the heavy neutrino  $N$  production, the cross section can be as large as a few  $fb$  for the mass scale as high as 1 TeV, higher than what can be achieved by the p-p collision mode of LHC with the same related heavy neutrino couplings.

## 2. Theoretical Models and Calculations

### 2.1 The Seesaw Type-I Model

In the ST-I model, the seesaw mechanism is realized by introducing right-handed neutrinos that are singlets under the SM gauge group  $SU(3)_C \times SU(2)_L \times U(1)_Y$  and can therefore have Majorana masses [11–16]. We assume for definiteness that there are three of these heavy neutrinos,  $N_{iR}$ , responsible for giving masses to the light neutrinos,  $\nu_{iL}$ . The Lagrangian describing the masses of the neutrinos can be written as

$$\mathcal{L} = -\bar{N}_{iR}(Y_D)_{ij}\tilde{H}^\dagger L_{jL} - \frac{1}{2}\bar{N}_{iR}(M_N)_{ij}N_{jR}^c + \text{H.c.}, \quad (2.1)$$

where summation over  $i, j = 1, 2, 3$  is implied.  $Y_D$  is the  $3 \times 3$  Yukawa coupling matrix.  $\tilde{H} = i\tau_2 H^*$  with  $\tau_2$  being the usual Pauli matrix.  $H = (\phi^+, (v+h+i\eta)/\sqrt{2})^T$  is the Higgs doublet and  $v$  its vacuum expectation value.  $L_{iL} = (\nu_{iL} \ l_{iL}^-)^T$  is the left-handed lepton doublet.  $M_N$  is the Majorana mass matrix, and  $N_{iR}^c$  is the charge conjugation of  $N_{iR}$ .

In the  $(\nu_L, N_R^c)^T$  basis the mass matrix is given by

$$M_{\text{seesaw}} = \begin{pmatrix} 0 & m_D^T \\ m_D & M_N \end{pmatrix}, \quad (2.2)$$

with the Dirac mass matrix  $m_D = vY_D/\sqrt{2}$ .

One can relate the weak eigenstates  $\nu_{iL}$  and  $N_{iR}^c$  to the corresponding mass eigenstates according to

$$\begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix} = U \begin{pmatrix} \nu_{mL} \\ N_{mL} \end{pmatrix}, \quad U \equiv \begin{pmatrix} V_{\nu\nu} & V_{\nu N} \\ V_{N\nu} & V_{NN} \end{pmatrix}. \quad (2.3)$$

where  $\nu_{mL}$  and  $N_{mL}$  are column matrices containing the mass eigenstates. Thus  $U$  is unitary and diagonalizes  $M_{\text{seesaw}}$ ,

$$\begin{pmatrix} \hat{m}_\nu & 0 \\ 0 & \hat{M}_N \end{pmatrix} = U^T M_{\text{seesaw}} U, \quad (2.4)$$

where  $\hat{m}_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$  and  $\hat{M}_N = \text{diag}(m_{N_1}, m_{N_2}, m_{N_3})$ . Note that the submatrices  $V_{\nu\nu}$ ,  $V_{\nu N}$ ,  $V_{N\nu}$ , and  $V_{NN}$  are not unitary.

In the mass eigenstate basis, dropping the subscript  $m$  for mass eigenstate fields, the gauge and Higgs interactions with  $N$  are given by

$$\begin{aligned}
\mathcal{L}_{CC} &= -\frac{g}{\sqrt{2}} \left( \bar{l}_L \gamma^\mu V_{PMNS} \nu_L W_\mu^- + \bar{l}_L \gamma^\mu V_{lN} N_L W_\mu^- + \text{H.c.} \right) \\
\mathcal{L}_{NC} &= \frac{g}{2c_W} \left( \bar{\nu}_L \gamma^\mu V_{PMNS}^\dagger V_{PMNS} \nu_L + \bar{N}_L \gamma^\mu V_{lN}^\dagger V_{PMNS} \nu_L \right. \\
&\quad \left. + \bar{\nu}_L \gamma^\mu V_{PMNS}^\dagger V_{lN} N_L + \bar{N}_L \gamma^\mu V_{lN}^\dagger V_{lN} N_L \right) Z_\mu \\
\mathcal{L}_S &= \frac{g}{2M_W} \left[ \bar{\nu}_L \hat{m}_\nu V_{PMNS}^\dagger V_{PMNS} \nu_L + \bar{N}_L^\dagger \hat{M}_N V_{lN}^\dagger V_{PMNS} \nu_L \right. \\
&\quad \left. + \bar{\nu}_L \hat{m}_\nu V_{PMNS}^\dagger V_{lN} N_L + \bar{N}_L^\dagger \hat{M}_N V_{lN}^\dagger V_{lN} N_L + \text{H.c.} \right] h
\end{aligned} \tag{2.5}$$

where  $g$  is the  $SU(2)_L$  coupling and  $c_W = \cos \theta_W$  with  $\theta_W$  being the Weinberg angle.  $V_{lN} = V_{\nu N}$ . The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix  $V_{PMNS} = V_{\nu\nu}$  is the usual light neutrino mixing matrix [45, 46]. Strictly speaking it is not unitary in seesaw models. But to the leading order the non-unitary effects can be neglected.

It is clear from the above expressions that the heavy neutrinos  $N_R$  can interact with the SM gauge bosons via mixing at the tree level. The leading parton level single heavy seesaw particle production at e-p collision is through

$$e + q \rightarrow N + q' \quad (\text{through t-channel } W\text{-exchange.}) \tag{2.6}$$

Combining the interaction Lagrangian in Eq.(2.5) and the charged quark current interaction,

$$\mathcal{L}' = -\frac{g}{\sqrt{2}} \left( \bar{u} \gamma^\mu V_{CKM} d W_\mu^+ + \text{H.c.} \right), \tag{2.7}$$

where  $V_{CKM}$  is the Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix [47, 48] for quarks, we obtain the corresponding amplitudes, for example,  $\mathcal{M}$  for  $e^-(p_1) + u(p_2) \rightarrow N_i(p_3) + d(p_4)$ ,

$$\begin{aligned}
\mathcal{M} &= i \frac{g^2}{2} (V_{lN})_{1i}^* V_{ud}^* \frac{g_{\mu\nu}}{\hat{t} - m_W^2} \\
&\quad \times \left[ \bar{N}_i \gamma^\mu P_L e \bar{d} \gamma^\nu P_L u + \frac{1}{m_W} \bar{N}_i (m_{N_i} P_L - m_e P_R) e \bar{d} (m_d P_L - m_u P_R) u \right],
\end{aligned} \tag{2.8}$$

where  $\hat{t} = (p_1 - p_3)^2$ . We obtain the t-channel  $W$ -exchange cross section for the partonic process  $e^-(p_1) + q(p_2) \rightarrow N_i(p_3) + q'(p_4)$  expressed as

$$\begin{aligned}
\sigma_W(e^- q \rightarrow N_i q') &= \int_{\hat{t}_0}^{\hat{t}_1} d\hat{t} \frac{g^4 |(V_{lN})_{1i}|^2}{64\pi \hat{s} p_{in}^2} \frac{1}{(\hat{t} - m_W^2)^2} \\
&\quad \times \begin{cases} |V_{qq'}|^2 (p_1 \cdot p_2) (p_3 \cdot p_4), & \text{if } q = u, c, q' = d, s, b. \\ |V_{q'\bar{q}}|^2 (p_1 \cdot p_4) (p_2 \cdot p_3), & \text{if } q = \bar{d}, \bar{s}, q' = \bar{u}, \bar{c}. \end{cases}
\end{aligned} \tag{2.9}$$

where

$$\begin{aligned}
\hat{s} &= (p_1 + p_2)^2, \\
p_{in}^2 &= \frac{(\hat{s} + m_2^2 - m_1^2)^2}{4\hat{s}} - m_2^2, \\
p_{out}^2 &= \frac{(\hat{s} + m_4^2 - m_3^2)^2}{4\hat{s}} - m_4^2, \\
\hat{t}_0 &= \frac{(m_1^2 - m_2^2 - m_3^2 + m_4^2)^2}{4\hat{s}} - (p_{in} + p_{out})^2, \\
\hat{t}_1 &= \frac{(m_1^2 - m_2^2 - m_3^2 + m_4^2)^2}{4\hat{s}} - (p_{in} - p_{out})^2.
\end{aligned} \tag{2.10}$$

## 2.2 The Seesaw Type-III Model

The ST-III model consists of, in addition to the SM particles, left-handed triplet leptons with zero hypercharge,  $\Sigma_L \sim (1, 3, 0)$  under  $SU(3)_C \times SU(2)_L \times U(1)_Y$  [17]. We write the component fields as

$$\Sigma_L = \begin{pmatrix} \Sigma_L^0/\sqrt{2} & \Sigma_L^+ \\ \Sigma_L^- & -\Sigma_L^0/\sqrt{2} \end{pmatrix}. \tag{2.11}$$

The charge conjugated form is

$$\Sigma_L^c = \begin{pmatrix} \Sigma_L^{0c}/\sqrt{2} & \Sigma_L^{-c} \\ \Sigma_L^{+c} & -\Sigma_L^{0c}/\sqrt{2} \end{pmatrix}. \tag{2.12}$$

Note that  $\Sigma_L^c$  is right-handed field.

The renormalizable Lagrangian involving  $\Sigma_L$  ( $\Sigma_L^c$ ) is given by

$$\mathcal{L} = \text{Tr}[\overline{\Sigma}_L i\gamma_\mu D^\mu \Sigma_L] - \frac{1}{2} \text{Tr}[\overline{\Sigma}_L^c M_\Sigma \Sigma_L + \overline{\Sigma}_L M_\Sigma^* \Sigma_L^c] - \overline{L}_L \sqrt{2} Y_\Sigma^\dagger \Sigma_L^c \tilde{H} - \tilde{H}^\dagger \overline{\Sigma}_L^c \sqrt{2} Y_\Sigma L_L. \tag{2.13}$$

One can define  $\Psi \equiv \Sigma_L^- + \Sigma_L^{+c}$  with  $\Psi_L = \Sigma_L^-$ ,  $\Psi_R = \Sigma_L^{+c}$  to obtain the charged and neutral lepton mass matrices in the basis,  $(l_L, \Psi_L)^T$  and  $(\nu_L, \Sigma_L^0)^T$ , respectively, as

$$\begin{pmatrix} m_l & 0 \\ vY_\Sigma & M_\Sigma \end{pmatrix}, \quad \begin{pmatrix} 0 & vY_\Sigma^T/2\sqrt{2} \\ vY_\Sigma/2\sqrt{2} & M_\Sigma/2 \end{pmatrix}. \tag{2.14}$$

The diagonalization of the mass matrices can be achieved by making unitary transformations on the triplet, the charged and neutral leptons defined in the following

$$\begin{pmatrix} l_{L,R} \\ \Psi_{L,R} \end{pmatrix} = U_{L,R} \begin{pmatrix} l_{mL,R} \\ E_{mL,R} \end{pmatrix}, \quad \begin{pmatrix} \nu_L \\ \Sigma_L^0 \end{pmatrix} = U_0 \begin{pmatrix} \nu_{mL} \\ N_{mL} \end{pmatrix}, \tag{2.15}$$

where  $U_{L,R}$  and  $U_0$  are  $6 \times 6$  unitary matrices which we decompose them into  $3 \times 3$  block matrices as

$$U_L \equiv \begin{pmatrix} U_{Lll} & U_{Ll\Psi} \\ U_{L\Psi l} & U_{L\Psi\Psi} \end{pmatrix}, \quad U_R \equiv \begin{pmatrix} U_{Rll} & U_{Rl\Psi} \\ U_{R\Psi l} & U_{R\Psi\Psi} \end{pmatrix}, \quad U_0 \equiv \begin{pmatrix} U_{0\nu\nu} & U_{0\nu\Sigma} \\ U_{0\Sigma\nu} & U_{0\Sigma\Sigma} \end{pmatrix}. \tag{2.16}$$

To the leading order, we have gauge and Higgs interaction terms involving heavy triplet leptons as [39–42]

$$\begin{aligned}
\mathcal{L}_{NC(A+Z)} &= e\bar{E}\gamma^\mu EA_\mu + gc_W\bar{E}\gamma^\mu EZ_\mu, \\
\mathcal{L}_{NCZ} &= \frac{g}{2c_W}[\bar{\nu}(V_{PMNS}^\dagger V_{lN}\gamma^\mu P_L - V_{PMNS}^T V_{lN}^*\gamma^\mu P_R)N + \sqrt{2}\bar{l}V_{lN}\gamma^\mu P_L E + \text{H.c.}]Z_\mu, \\
\mathcal{L}_{CC} &= -g[\bar{E}\gamma^\mu N + \frac{1}{\sqrt{2}}\bar{l}V_{lN}\gamma^\mu P_L N + \bar{E}V_{lN}^T V_{PMNS}^*\gamma^\mu P_R \nu]W_\mu^- + \text{H.c.}, \\
\mathcal{L}_S &= -\frac{g}{2M_W}[\bar{\nu}(V_{PMNS}^\dagger V_{lN}\hat{M}_N P_R + V_{PMNS}^T V_{lN}^*\hat{M}_N P_L)N + \sqrt{2}\bar{l}V_{lN}\hat{M}_E P_R E]h + \text{H.c.},
\end{aligned} \tag{2.17}$$

with  $V_{lN} \equiv V_{l\Sigma}^L = -Y_\Sigma^\dagger v M_\Sigma^{-1}/\sqrt{2}$ . In the above, all fields are in mass eigenstates.  $\hat{M}_E, \hat{M}_N$  are eigen-mass matrices of  $E, N$ .

In this model, there is also the possibility of producing the heavy charged leptons,  $E$ . The heavy seesaw particle production at the e-p collider involves the following partonic processes,

$$\begin{aligned}
e^- q &\rightarrow N q' \quad (\text{through t-channel } W\text{-exchange.}), \\
e^- q &\rightarrow E^- q \quad (\text{through t-channel } Z\text{- and } h\text{-exchange.})
\end{aligned} \tag{2.18}$$

The cross section,  $\sigma_W(e^- q \rightarrow N q')$ , has the same expression as that given in Eq.(2.9). The t-channel  $Z$ - and  $h$ -exchange for  $E$  production cross section,  $\sigma_{Zh}(e^- q \rightarrow E^- q)$ , can be obtained using the interactions in Eq.(2.17). The contribution from  $h$ -exchange involves a small light quark-Higgs Yukawa coupling and therefore is small. For the  $Z$ -exchange contribution, we denote the processes as  $e^-(p_1) + q(p_2) \rightarrow E_i^-(p_3) + q(p_4)$  and have

$$\begin{aligned}
\sigma_{Zh}(e^- q \rightarrow E_i^- q) &= \int_{\hat{t}_0}^{\hat{t}_1} d\hat{t} \frac{g^4 |(V_{lN})_{li}|^2}{32\pi c_W^4 \hat{s} p_{in}^2} \frac{1}{(\hat{t} - m_Z^2)^2} \\
&\times \begin{cases} [g_L^2(q)(p_1 \cdot p_2)(p_3 \cdot p_4) + g_R^2(q)(p_1 \cdot p_4)(p_2 \cdot p_3)], & \text{if } q = u, c, d, s. \\ [g_R^2(q)(p_1 \cdot p_2)(p_3 \cdot p_4) + g_L^2(q)(p_1 \cdot p_4)(p_2 \cdot p_3)], & \text{if } q = \bar{u}, \bar{c}, \bar{d}, \bar{s}. \end{cases}
\end{aligned} \tag{2.19}$$

where  $\hat{s}, \hat{t}, p_{in}, \hat{t}_0$  and  $\hat{t}_1$  have the same definitions as in Eqs.(2.9)-(2.10), and

$$\begin{aligned}
g_L(u) &= g_L(c) = g_L(\bar{u}) = g_L(\bar{c}) = \frac{1}{2} - \frac{2}{3}s_W^2, \\
g_R(u) &= g_R(c) = g_R(\bar{u}) = g_R(\bar{c}) = -\frac{2}{3}s_W^2, \\
g_L(d) &= g_L(s) = g_L(\bar{d}) = g_L(\bar{s}) = -\frac{1}{2} + \frac{1}{3}s_W^2, \\
g_R(d) &= g_R(s) = g_R(\bar{d}) = g_R(\bar{s}) = \frac{1}{3}s_W^2.
\end{aligned} \tag{2.20}$$

### 3. Numerical analysis

#### 3.1 Production of $N$ and $E$ at the LHeC

To obtain the cross sections for  $e^- p \rightarrow N_1 j + X$  process ( $j$  indicates a hard quark jet) in the ST-I and ST-III models, and for the  $e^- p \rightarrow E_1^- j + X$  process in the ST-III model, one

$m_{N_1}(m_{E_1})$ (GeV)	$\sigma(e^-p \rightarrow N_1j + X)/ (V_{lN})_{11} ^2$ (pb)		$\sigma(e^-p \rightarrow E_1^-j + X)/ (V_{lN})_{11} ^2$ (pb)	
	$E_e = 70\text{GeV}$	$E_e = 140\text{GeV}$	$E_e = 70\text{GeV}$	$E_e = 140\text{GeV}$
100	147.8	211.1	103.4	153.7
200	75.15	117.2	49.71	81.89
400	22.67	46.29	13.46	29.63
600	6.049	19.46	3.27	11.55
800	1.125	7.614	0.552	4.22
1000	0.1028	2.549	0.0448	1.32

**Table 1:** The numerical results for the cross sections for  $e^-p \rightarrow N_1j + X$  in the ST-I and ST-III models, and  $e^-p \rightarrow E_1^-j + X$  in the ST-III model at the LHeC as the functions of  $m_{N_1}$  and  $m_{E_1}$ , respectively. There we take  $E_p = 7$  TeV.

needs to fold in the parton distributions (PDFs) of the proton. We use CTEQ6L1 PDFs for the LO calculations.

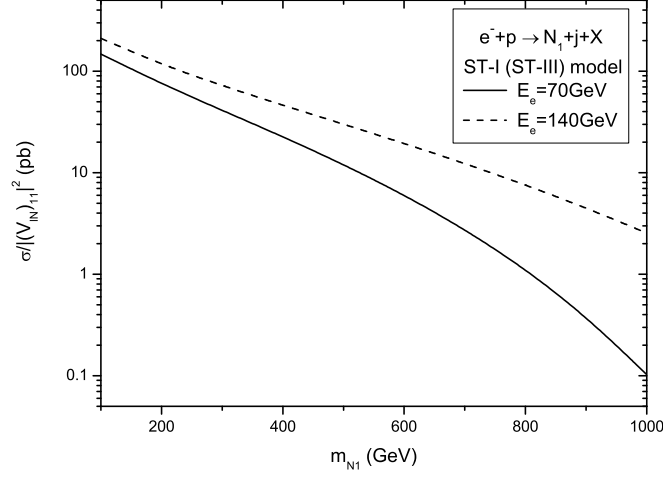
The calculations for the processes  $e^-p \rightarrow N_1j + X$  and  $e^-p \rightarrow E_1^-j + X$  involve the contributions of the partonic processes  $e^-q \rightarrow N_1q'$  (where  $q = u, d, c, s$  and  $q' = u, d, s, c, b$ ), and  $e^-q \rightarrow E_1^-q$  (where  $q = u, d, c, s$ ), respectively. We neglect the masses of electron and light quarks  $u, d, s$ , and assume  $m_h = 115$  GeV. For the CKM quark mixing parameters, we will use central values  $\lambda = 0.2257$ ,  $A = 0.814$ ,  $\bar{\rho} = 0.135$  and  $\bar{\eta} = 0.349$  in Ref. [1] to determine the angles and phase in the Particle Data Group (PDG) parametrization of CKM matrix and use them in our numerical calculations. We have,  $s_{12} = 0.2257$ ,  $s_{23} = 0.0415$ ,  $s_{13} = 0.00359$  and  $\delta = 68.88^\circ$ . The other input parameters involved are taken as [1]:

$$\begin{aligned} \alpha^{-1}(m_Z^2) &= 127.918, \quad m_Z = 91.1876 \text{ GeV}, \quad m_W = 80.398 \text{ GeV}, \\ m_c &= 1.3 \text{ GeV}, \quad m_b = 4.2 \text{ GeV}. \end{aligned} \quad (3.1)$$

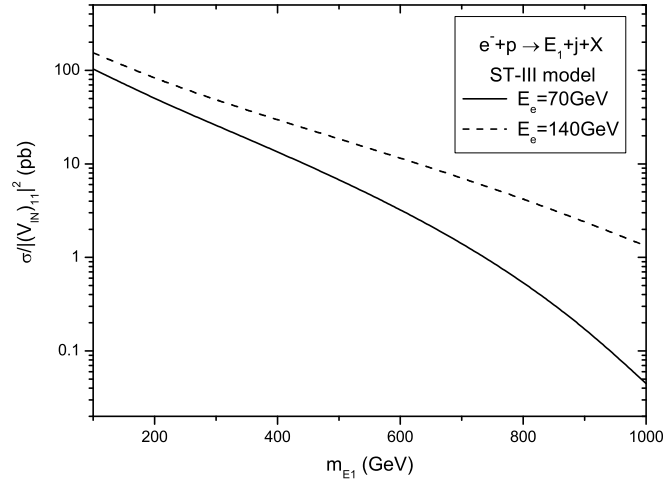
As a demonstration of numerical results we take  $E_p = 7$  TeV,  $E_e = 70$  GeV or 140 GeV, and set the factorization scales  $\mu_f = \frac{1}{2}m_{N_1}$  for process  $e^-p \rightarrow N_1j + X$ , and  $\mu_f = \frac{1}{2}m_{E_1}$  for process  $e^-p \rightarrow E_1^-j + X$ .

In Fig.1 and Fig.2, we depict the production rates of the processes  $e^-p \rightarrow N_1j + X$  in the ST-I and ST-III models and  $e^-p \rightarrow E_1^-j + X$  in the ST-III model as functions of the masses of  $N_1$  and  $E_1^-$ , respectively. For the  $N_1j$  production process the  $\sigma(e^-p \rightarrow N_1j + X)/|(V_{lN})_{11}|^2$  results in both the ST-I and ST-III models are the same. In Table 1 we also list some of the representative numerical results which are read off from Fig.1 and Fig.2.

Whether it is possible at the LHeC to test the ST-I and ST-III models not only depends on the masses of the heavy states, but also crucially depends on how large  $V_{lN}$  can be in order to have sufficient number of  $N_i$  and  $E_i^-$  to be produced. If there is only one generation, one would obtain  $V_{lN}$  of order  $m_D/m_N$  whose magnitude is  $\sqrt{|m_\nu/m_N|}$ . With light neutrino mass constrained to be less than of order 1 eV, the magnitude of mixing  $V_{lN}$  is



**Figure 1:** The cross sections for  $N_1 j$  production at the LHeC as the functions of  $m_{N_1}$  with  $E_p = 7$  TeV, where the full-line is for  $E_e = 70$  GeV and the dashed-line for  $E_e = 140$  GeV.



**Figure 2:** The cross sections for  $E_1^- j$  production at the LHeC as the functions of  $m_{E_1}$  with  $E_p = 7$  TeV, where the full-line is for  $E_e = 70$  GeV and the dashed-line for  $E_e = 140$  GeV.

bounded by  $10^{-6} \sqrt{100 \text{ GeV} / m_N}$ . Even with  $m_N$  of order 100 GeV, the mixing is extremely small. With such a small mixing it is not possible to produce enough number of heavy neutrinos to study its properties at the LHeC. However, this conclusion is true only for one generation of neutrino. With more than one generation, one can evade the constraint



$|V_{lN}| = \sqrt{|m_\nu/m_N|}$ . There are solutions of the form  $V_{lN} = V_0 + V_\delta$  satisfying constraints on light neutrino masses and mixing with some of the elements in  $V_0$  to be left unconstrained from neutrino masses [22, 49–58], as long as  $V_0 \hat{M}_N V_0^T = 0$  which requires that  $V_0$  being rank one [58]. Such solutions have interesting implications [59–61]. Constraints on the sizes of elements in  $\epsilon = V_{lN} V_{lN}^\dagger$  then come from flavor changing neutral current processes. For the ST-I model, we have the constraints as [38, 62, 63]

$$\epsilon_{11} \leq 3.0 \times 10^{-3}, \quad \epsilon_{22} \leq 3.2 \times 10^{-3}, \quad \epsilon_{33} \leq 6.2 \times 10^{-3}, \quad (3.2)$$

and Refs. [62–64] give the limitations as

$$|\epsilon_{12}| \leq 1 \times 10^{-4}, \quad |\epsilon_{13}| \leq 0.01, \quad |\epsilon_{23}| \leq 0.01. \quad (3.3)$$

For heavy neutrinos coupling to the electron, neutrinoless double-beta decay imposes [65]

$$\left| \sum_{i=1}^3 (V_{lN})_{1i}^2 / m_{N_i} \right| \leq 5 \times 10^{-8} \text{ GeV}^{-1}. \quad (3.4)$$

Finally, there are also constraints from searches for SM-singlet neutrinos by the L3 and DELPHI experiments at LEP [66, 67] on the individual elements  $(V_{lN})_{2i}$  and  $(V_{lN})_{3i}$  which may be stronger than those inferred from Eqs. (3.2) and (3.3), depending on  $m_{N_i}$ .

For the ST-III model, because additional  $E^\pm$  exist in the model the constraints are slightly stronger. They are [38, 62, 63]

$$\epsilon_{11} \leq 3.6 \times 10^{-4}, \quad \epsilon_{22} \leq 2.9 \times 10^{-4}, \quad \epsilon_{33} \leq 7.3 \times 10^{-4}, \quad (3.5)$$

whereas from the measurements of lepton-flavor violating transitions [68]

$$|\epsilon_{12}| \leq 1.7 \times 10^{-7}, \quad |\epsilon_{13}| \leq 4.2 \times 10^{-4}, \quad |\epsilon_{23}| \leq 4.9 \times 10^{-4}. \quad (3.6)$$

In addition, direct searches for heavy charged leptons at colliders impose constraints on the mass of  $E$ , and hence the mass of  $N$  as well, namely  $m_{N_i}, m_{E_i} \gtrsim 100 \text{ GeV}$ .

If the heavy neutrinos are nearly degenerate, we can get the relation for the cross sections from Eq.(2.9) expressed as

$$\begin{aligned} & \sigma(e^- p \rightarrow N_1 j + X) : \sigma(e^- p \rightarrow N_2 j + X) : \sigma(e^- p \rightarrow N_3 j + X) \\ & = |(V_{lN})_{11}|^2 : |(V_{lN})_{12}|^2 : |(V_{lN})_{13}|^2, \end{aligned} \quad (3.7)$$

From the definition of  $\epsilon$  matrix we have  $\epsilon_{11} = \sum_{i=1}^3 |(V_{lN})_{1i}|^2$ . Then we obtain

$$\begin{aligned} \sum_{i=1}^3 \sigma(e^- p \rightarrow N_i j + X) &= \sum_{i=1}^3 |(V_{lN})_{1i}|^2 \sigma(e^- p \rightarrow N_1 j + X) / |(V_{lN})_{11}|^2 \\ &= \epsilon_{11} \sigma(e^- p \rightarrow N_1 j + X) / |(V_{lN})_{11}|^2. \end{aligned} \quad (3.8)$$

Analogously the relations of Eqs.(3.7) and (3.8) are also satisfied by the production processes  $e^- p \rightarrow E_i^- j + X$  with the nearly degenerate heavy charged lepton  $E_i^-$ .

From Fig.1 and Fig.2, one can read off the cross sections for  $N$  and  $E$  productions with appropriate constraints. Eq.(3.4) gives the strongest constraint on the couplings. Using that constraint literally, it is not possible to have large enough events produced. However, note that the couplings in Eq.(3.4) involve a summation of terms not necessarily positively defined. If one can reconstruct the  $N_i$  mass eigenstates which may be able to do with sufficient mass splitting, the production cross sections will be proportional to the square of the absolute value of each term. There are chances of cancelation between terms [58] and leave individual terms sizeable. Such cancelation occurrence has to be determined by experiments. We will discuss this more later. Here we will assume that this is the case, and then take the upper bounds  $\epsilon_{11} = 3 \times 10^{-3}$ ,  $3.6 \times 10^{-4}$  for the ST-I and ST-III models, respectively, for discussions. From the figures we see that for  $E_e = 70$  GeV, the cross section for  $N$  production in the ST-I model, can be as high as a few fb with mass as large as 700 GeV. For  $E_e = 140$  GeV, even with a mass scale as large as a TeV the cross section can be more than a fb. These cross sections are much higher than that can be achieved for single  $N$  production at the LHC. With an integrated luminosity of  $100 \text{ fb}^{-1}$ , there are several hundred events can be produced and analyzed. For the ST-III model, the event number is smaller. But can still have hundreds of events for the mass scale to be as high as 800 GeV with  $E_e = 140$  GeV. In this case there is also the possibility of studying  $E$  production. The mass scale can be probed up 800 GeV too.

### 3.2 Signals of $N$ and $E$

The identification of the productions of  $N$  and  $E$  can be studied by reconstructing decay products of  $N$  and  $E$ . In order to have large enough production cross sections, the parameter  $(V_{lN})_{ij}$  need to be close to their allowed bounds. With these bounds,  $N$  and  $E$  will decay inside the detector. The final states will involve a lepton and a  $W$  or  $Z$  from  $N$  or  $E$  decays, and it would be most convenient to choose the charged lepton in the final states to analyze. To keep the analysis as simple as possible, one can reconstruct the final  $W$  and  $Z$  using their hadronic decay modes. This procedure will reduce the available events for analysis depending on the branching ratios of  $N$  and  $E$  decays. We now provide some details.

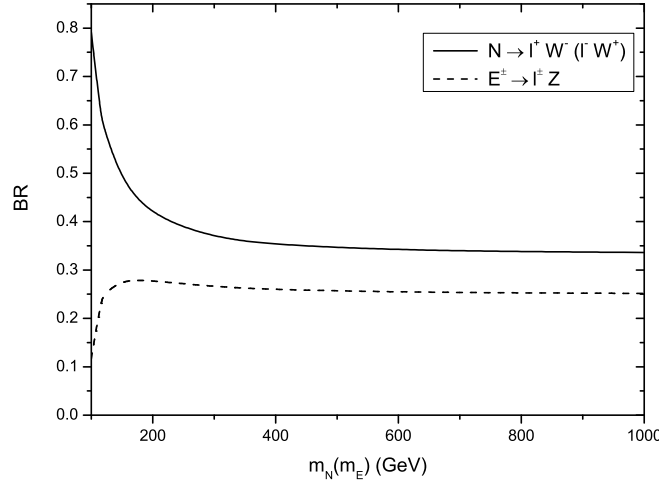
In the ST-I model, the dominant decay modes for  $N$  are

$$\begin{aligned}\Gamma(N_i \rightarrow \ell^- W^+) &= \Gamma(N_i \rightarrow \ell^+ W^-) = \frac{g^2}{64\pi} |(V_{lN})_{\ell i}|^2 \frac{m_{N_i}^3}{m_W^2} \left(1 - \frac{m_W^2}{m_{N_i}^2}\right) \left(1 + \frac{m_W^2}{m_{N_i}^2} - 2\frac{m_W^4}{m_{N_i}^4}\right), \\ \sum_{m=1}^3 \Gamma(N_i \rightarrow \nu_m Z^0) &= \frac{g^2}{64\pi c_W^2} \sum_{\ell=1}^3 |(V_{lN})_{\ell i}|^2 \frac{m_{N_i}^3}{m_Z^2} \left(1 - \frac{m_Z^2}{m_{N_i}^2}\right) \left(1 + \frac{m_Z^2}{m_{N_i}^2} - 2\frac{m_Z^4}{m_{N_i}^4}\right), \\ \sum_{m=1}^3 \Gamma(N_i \rightarrow \nu_m h^0) &= \frac{g^2}{64\pi} \sum_{\ell=1}^3 |(V_{lN})_{\ell i}|^2 \frac{m_{N_i}^3}{m_W^2} \left(1 - \frac{m_h^2}{m_{N_i}^2}\right)^2.\end{aligned}\tag{3.9}$$

In the above equations, we have assumed that  $V_{PMNS}$  is unitary. For the ST-III model, the neutral heavy neutrino  $N$  has the same form as that in the ST-I model given above,

while for the heavy charged lepton  $E$  decay widths are given by

$$\begin{aligned}
\sum_{m=1}^3 \Gamma(E_i^+ \rightarrow \bar{\nu}_m W^+) &= \frac{g^2}{32\pi} \sum_{\ell=1}^3 |(V_{lN})_{\ell i}|^2 \frac{m_{E_i}^3}{m_W^2} \left(1 - \frac{m_W^2}{m_{E_i}^2}\right) \left(1 + \frac{m_W^2}{m_{E_i}^2} - 2\frac{m_W^4}{m_{E_i}^4}\right), \\
\Gamma(E_i^+ \rightarrow \ell^+ Z^0) &= \frac{g^2}{64\pi c_W^2} |(V_{lN})_{\ell i}|^2 \frac{m_{E_i}^3}{m_Z^2} \left(1 - \frac{m_Z^2}{m_{E_i}^2}\right) \left(1 + \frac{m_Z^2}{m_{E_i}^2} - 2\frac{m_Z^4}{m_{E_i}^4}\right), \\
\Gamma(E_i^+ \rightarrow \ell^+ h^0) &= \frac{g^2}{64\pi} |(V_{lN})_{\ell i}|^2 \frac{m_{E_i}^3}{m_W^2} \left(1 - \frac{m_h^2}{m_{E_i}^2}\right)^2.
\end{aligned} \tag{3.10}$$



**Figure 3:** The branching ratios for  $N \rightarrow l^\pm W^\mp$  and  $E^\pm \rightarrow l^\pm Z$  as functions of  $m_N$  and  $m_E$ .

In Fig.3, we show the branching ratio for  $N \rightarrow l^\pm W^\mp$  and  $E^\pm \rightarrow l^\pm Z$  as functions of heavy seesaw mass. We see that the branching ratios are of order 33% and 25% when the heavy neutrino and heavy charged lepton masses are large enough, for example above 200 GeV.

Combining the hadronic decay branching ratios for  $W^\mp$  and  $Z$  are 67.6% and 69.91%, we find that although there are reduction factors, but there are still 22% and 17% of the produced  $N$  and  $E$  can be analyzed by  $N \rightarrow l^\pm + W^\mp$  and  $E^\pm \rightarrow l^\pm + Z$  followed by  $W$  and  $Z$  decay into hadronic final states, respectively.

### 3.3 The process $e^- p \rightarrow l^\pm W^\mp j + X$

There are two types of processes,  $e^- p \rightarrow l^+ W^- j + X$  and  $e^- p \rightarrow l^- W^+ j + X$ , which can be used to study the heavy neutrinos  $N_i$  ( $i = 1, 2, 3$ ). The former type also contains information about the Majorana nature of  $N_i$ .

After a straightforward calculation, we find that, for degenerate heavy neutrinos, the cross sections for the processes  $e^-p \rightarrow l^+W^-j + X$  with  $l = e, \mu, \tau$  are proportional to  $|\kappa_{11}|^2$ ,  $|\kappa_{12}|^2$  and  $|\kappa_{13}|^2$ , respectively, where the  $\kappa$  matrix is defined as

$$\kappa = V_{lN}^* V_{lN}^\dagger. \quad (3.11)$$

Analogously, the cross sections for the processes  $e^-p \rightarrow l^-W^+j + X$  ( $l = e, \mu, \tau$ ) are proportional to  $|\epsilon_{11}|^2$ ,  $|\epsilon_{12}|^2$  and  $|\epsilon_{13}|^2$  with the definition of

$$\epsilon = V_{lN} V_{lN}^\dagger. \quad (3.12)$$

Being a lepton number conserving process,  $e^-p \rightarrow e^-W^+j + X$  has large SM background, therefore, may be difficult to analyze. The process  $e^-p \rightarrow e^+W^-j + X$  which violates lepton number by two units may be more interesting to study. However, if the heavy neutrinos are indeed nearly degenerate and one cannot separately identify whether  $e^+$  is from which  $N_i$ , the constraint from the neutrinoless double-beta decay (Eq.(3.4)) implies that

$$|\kappa_{11}| \leq 5 \times 10^{-5} \times \frac{m_N}{1000 \text{ GeV}}, \quad (3.13)$$

then the cross section for the process  $e^-p \rightarrow e^+W^-j + X$  is too small to be measured. In this case the cross section for  $e^-p \rightarrow e^+W^-j + X$  is too small to be measured.

If the heavy neutrinos have sufficiently large mass splitting, one can reconstruct the individual  $N_i$  state by the invariant mass of  $e^+W^-$ , then one can have  $e^-p \rightarrow e^+W^-j + X$  with three different invariant mass of  $e^+W^-$ . The coherent cancelation in the degenerate case would not happen with each of the cross section proportional to the product of  $|(V_{lN})_{li}|^2$  and the  $N_i$  branching ratio into  $e^+$ . To this end we note that  $|(V_{lN})_{li}|^2$  much larger than  $|\kappa_{11}|$  is possible by noticing the following: In the degenerate heavy neutrino case, the constraint in Eq.(3.4) is proportional to  $\kappa_{11}$ . But for non-degenerate case, using  $V_{lN} \hat{M}_N V_{lN}^T \approx 0$  the condition for large  $V_{lN}$  solution, one can have solutions such that the combination of  $V_{lN}$  and  $m_{N_i}$  in Eq.(3.4) is automatically equal to zero if  $(m_{N_1}^2 - m_{N_2}^2)m_{N_3}/(m_{N_1}^2 - m_{N_3}^2)m_{N_2} = -(V_{lN})_{13}^2/(V_{lN})_{12}^2$ . The mass splitting and the couplings are therefore not separately constrained leaving the possibility of large mass splitting and couplings. In this case, one can then combine constraints  $\epsilon_{11} < 3 \times 10^{-3}$  and  $\epsilon_{11} < 3.6 \times 10^{-4}$ , for ST-I and ST-III, respectively, to estimate the production cross section.

Another new class of lepton number violating processes which are most promising to study at the LHeC are  $e^-p \rightarrow l^\pm W^\mp j + X$  ( $l = \mu, \tau$ ) channels with  $W$  decaying into hadron jets  $j_W$ . All these processes violate generation lepton number. For  $l^+$  the lepton number is violated by two units. The detection of this class of processes therefore can also reveal the Majorana nature of the heavy leptons. Since the SM backgrounds for these processes are small, they provide the opportunity to verify the existence of heavy neutrinos  $N_{1,2,3}$  cleanly without the possible stringent constraint from neutrinoless double-beta decay even the heavy neutrinos are degenerate.

Considering the fact that the upper bounds of  $|\epsilon_{12}|$  and  $|\epsilon_{13}|$  are relatively large (see Eq.(3.3) for the ST-I model and Eq.(3.6) for the ST-III model), the processes

$$e^-p \rightarrow N_{1,2,3} + j + X \rightarrow l^-W^+ + j + X \rightarrow l^-j_W + j + X, \quad (l = \mu, \tau) \quad (3.14)$$

$m_N$ (GeV)	ST-I model			ST-III model		
	$\Gamma(N)$ (GeV)	$\sigma(e^-p \rightarrow \tau^-W^+j + X) (fb)$		$\Gamma(N)$ (GeV)	$\sigma(e^-p \rightarrow \tau^-W^+j + X) (fb)$	
		$E_e = 70\text{GeV}$	$E_e = 140\text{GeV}$		$E_e = 70\text{GeV}$	$E_e = 140\text{GeV}$
100	$1.017 \times 10^{-3}$	1413	2017	$1.132 \times 10^{-4}$	22.40	31.98
200	0.05089	372.7	580.5	0.005663	5.901	9.199
400	0.5081	96.30	196.7	0.05655	1.530	3.114
600	1.773	25.08	80.29	0.1974	0.3952	1.2742
800	4.251	4.744	31.24	0.4731	0.07312	0.4928
1000	8.345	0.5709	10.67	0.9287	0.00689	0.1645

**Table 2:** The numerical results of the cross sections for the process  $e^-p \rightarrow \tau^-W^+j + X$  and the total decay widths of the heavy neutrinos by taking  $|\epsilon_{13}| = 0.01$  for the ST-I model and  $|\epsilon_{13}| = 4.2 \times 10^{-4}$  for the ST-III model, respectively.  $\Gamma(N)$  and  $\sigma$  represent the decay width of  $N$  and the cross section for the process  $e^-p \rightarrow \tau^-W^+j + X$ , respectively. There we take  $E_p = 7 \text{ TeV}$ .

at the LHeC may be practical.

The processes  $e^-p \rightarrow N_{1,2,3} + j + X \rightarrow l^+W^- + j + X \rightarrow l^+j_W + j + X$ , ( $l = \mu, \tau$ ) will be proportional to  $|\kappa_{12}|$  and  $|\kappa_{13}|$ . Unlike  $\kappa_{11}$ , there are no cancelations in  $\kappa_{12}$  and  $\kappa_{13}$ , in general. Therefore, they can have the same order of magnitude as that for  $\epsilon_{12}$  and  $\epsilon_{13}$  leading to similar cross sections as that of  $e^-p \rightarrow N_{1,2,3} + j + X \rightarrow l^-W^+ + j + X \rightarrow l^-j_W + j + X$ . In the following we will concentrate on  $l^-$  cases.

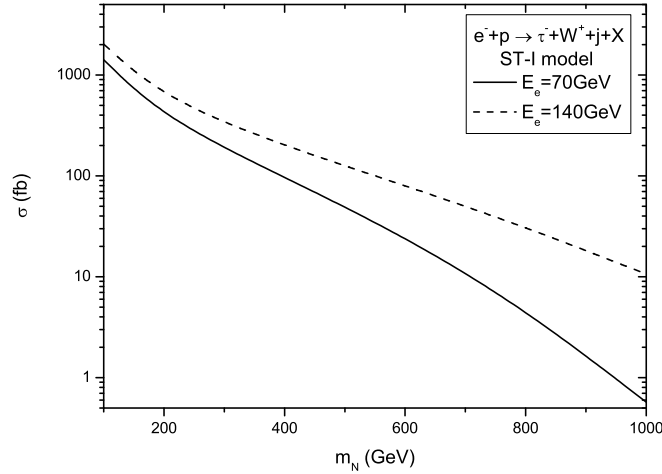
If we assume that the heavy neutrinos of three generations are quasi-degenerated and have the same mass ( $m_N$ ) and decay width, we get the relation of  $\sigma(e^-p \rightarrow \mu^-W^+j + X) : \sigma(e^-p \rightarrow \tau^-W^+j + X) = |\epsilon_{12}|^2 : |\epsilon_{13}|^2$ . In our numerical calculations, we take the upper bounds  $|\epsilon_{12}| = 1 \times 10^{-4}$ ,  $|\epsilon_{13}| = 0.01$  for the ST-I model and  $|\epsilon_{12}| = 1.7 \times 10^{-7}$ ,  $|\epsilon_{13}| = 4.2 \times 10^{-4}$  for the ST-III model, respectively.

In Table 2, We list some of the numerical results of the cross sections for the process  $e^-p \rightarrow \tau^-W^+j + X$  and total decay widths of heavy neutrinos with different values of  $m_N$  in both the ST-I and ST-III models. The total decay widths of heavy neutrinos could be obtained from their dominant decay modes:  $N_{1,2,3} \rightarrow l^+W^-, l^-W^+, \nu Z, \bar{\nu}Z, \nu h, \bar{\nu}h$ . We can see from the table that the cross section for the  $e^-p \rightarrow \tau^-W^+j + X$  process can reach few hundreds of  $fb$  when the heavy neutrino mass is several hundred GeV. Since  $|\epsilon_{12}|^2/|\epsilon_{13}|^2 \sim 10^{-4} - 10^{-7}$ , and  $\sigma(e^-p \rightarrow \tau^-W^+j + X) < 2.02 \times 10^3 \text{ fb}$  (see Table 2), we obtain that the production cross sections for  $e^-p \rightarrow \mu^-W^+j + X$  process in the ST-I and ST-III models are  $10^{-4}$  and  $10^{-7}$  smaller than for the  $\tau^-W^+j$  production process respectively, and

$$\sigma(e^-p \rightarrow \mu^-W^+j + X) = \sigma(e^-p \rightarrow \tau^-W^+j + X) \times \frac{|\epsilon_{12}|^2}{|\epsilon_{13}|^2} < 2.02 \times 10^{-1} \text{ fb}. \quad (3.15)$$

Therefore, we can conclude that  $e^-p \rightarrow \tau^-W^+j + X$  is an ideal signal process for search the heavy neutrinos, but the  $e^-p \rightarrow \mu^-W^+j + X$  process is not, due to its small cross section.

In Fig.4 and Fig.5, we depict the cross sections for the process  $e^-p \rightarrow \tau^- W^+ j + X$  at the LHeC as functions of the mass of  $N$  in the ST-I and ST-III models, respectively. For  $E_e = 140$  GeV in the ST-I model, with  $100 \text{ fb}^{-1}$  integrated luminosity, there are over one thousand events for the lepton number violating process study for a seesaw mass scale of 1 TeV. Lowering  $E_e$  to 70 GeV, the reach of the seesaw mass scale is also lowered, but can still have more than a thousand events to be studied for seesaw mass scalae as high as 700 GeV. For the ST-III model, more than a hundred events will be accumulated when  $m_N < 700$  and 400 GeV for the case of  $E_e = 140$  and 70 GeV, respectively.

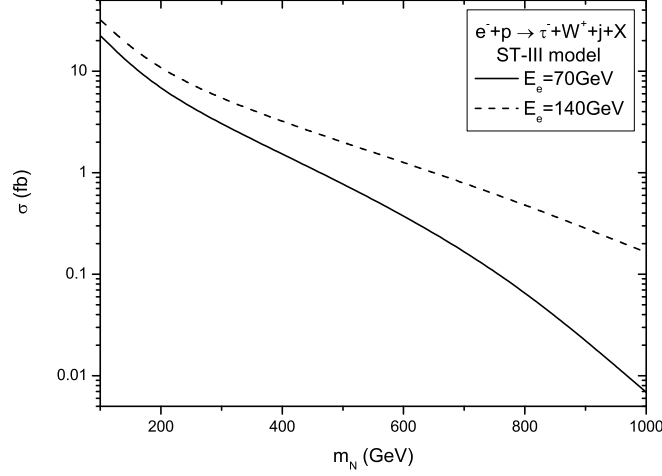


**Figure 4:** The cross sections for the  $\tau^- W^+ j$  production in the ST-I model at the LHeC as functions of  $m_N$  with  $E_p = 7$  TeV and  $|\epsilon_{13}| = 0.01$ , where the full-line is for  $E_e = 70$  GeV and the dashed-line for  $E_e = 140$  GeV.

### 3.4 The process $e^-p \rightarrow l^- Z j + X$

Analogous to the  $N$  production,  $e^-p \rightarrow E_{1,2,3}^- + j + X \rightarrow l^- Z + j + X \rightarrow l^- j_Z + j + X$  with  $l = \mu, \tau$  are lepton number violating processes, therefore, might be signal processes for the search of the heavy charged leptons  $E_i$  ( $i = 1, 2, 3$ ). But the  $e^-p \rightarrow E_{1,2,3}^- j + X \rightarrow e^- Z + j + X \rightarrow e^- j_Z + j + X$  channel can not be an ideal signal process for testing the type-III seesaw model because of the large SM background.

Since the experimental upper bound for  $|\epsilon_{12}|$  is  $1.7 \times 10^{-7}$  which is much smaller than that for  $|\epsilon_{13}|$  in the ST-III model, only the  $\tau^- Z j$  production rate with interchanging heavy charged leptons  $e^-p \rightarrow E_{1,2,3}^- j + X \rightarrow \tau^- Z j + X$  is significant. We assume all the heavy charged leptons of three generations are quasi-degenerated and have the same mass ( $m_E$ ) and decay width, and take  $|\epsilon_{13}| = 4.2 \times 10^{-4}$  in the ST-III model. In Fig.6 we present the  $m_E$  dependence of the cross section for the  $\tau^- Z j$  production at the LHeC, where the full-line is for  $E_e = 70$  GeV and the dashed-line for  $E_e = 140$  GeV. Some of the numerical



**Figure 5:** The cross sections for the  $\tau^- W^+ j$  production in the ST-III model at the LHeC as functions of  $m_N$  with  $E_p = 7$  TeV and  $|\epsilon_{13}| = 4.2 \times 10^{-4}$ , where the full-line is for  $E_e = 70$  GeV and the dashed-line for  $E_e = 140$  GeV.

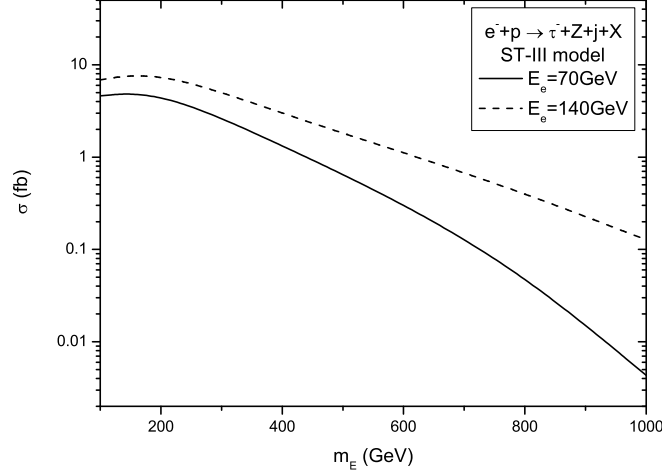
$m_E$ (GeV)	$\Gamma(E)$ (GeV)	$\sigma(e^- p \rightarrow \tau^- Z j + X)$ (fb)	
		$E_e = 70\text{GeV}$	$E_e = 140\text{GeV}$
100	$1.014 \times 10^{-4}$	4.622	6.877
200	0.003993	5.297	8.731
400	0.03822	1.337	2.944
600	0.1324	0.3187	1.126
800	0.3165	0.05343	0.4077
1000	0.6205	0.00435	0.1270

**Table 3:** The numerical results of the cross sections for the process  $e^- p \rightarrow \tau^- Z j + X$  and the total decay widths of the heavy charged leptons in the ST-III model.  $\Gamma(E)$  and  $\sigma$  represent the decay width of  $E$  and the cross section for the process  $e^- p \rightarrow \tau^- Z j + X$ , respectively. There we take  $E_p = 7$  TeV and  $|\epsilon_{13}| = 4.2 \times 10^{-4}$ .

results in Fig.6 are listed in Table 3. We can see from these results that the cross section for the  $e^- p \rightarrow E_{1,2,3}^- j \rightarrow \tau^- Z j + X$  process can reach a few fb when the heavy charged lepton mass  $m_E < 600$  and 400 GeV for  $E_e = 140$  and 70 GeV, respectively.

#### 4. Conclusions

We have studied the potential of testing the seesaw type I and III models at the LHeC. The e-p collision mode provides an excellent place to study lepton number violating processes



**Figure 6:** The cross sections for the  $\tau^- Z j$  production in the ST-III model at the LHeC as functions of  $m_E$  with  $E_p = 7$  TeV and  $|\epsilon_{13}| = 4.2 \times 10^{-4}$ , where the full-line is for  $E_e = 70$  GeV and the dashed-line for  $E_e = 140$  GeV.

$e^- p \rightarrow N_i j + X \rightarrow e^+ W^- j + X$  and  $e^- p \rightarrow N_i j + X \rightarrow \tau^- W^+ j + X$  with  $W$  into hadron jets. Here  $N_{1,2,3}$  are heavy Majorana neutrinos and  $j$  is a hard hadron jet. Although the process  $e^- p \rightarrow N_i j + X \rightarrow e^+ W^- j + X$  is stringently constrained from neutrinoless double-beta decay, there are solutions where this constraint can be satisfied with sizeable production cross section. For the process  $e^- p \rightarrow N_i j + X \rightarrow \tau^\pm W^\mp j + X$ , the neutrinoless double-beta decay constraint does not apply. With  $E_e = 140$  GeV and  $E_p = 7$  TeV, we find that the cross section for the heavy neutrino  $N$  can be higher than what can be achieved by the p-p collision mode of LHC with same related heavy neutrino couplings. For  $E_e = 140$  GeV, with  $100 \text{ fb}^{-1}$  integrated luminosity, there are over one thousand events for the lepton number violating process study for a seesaw mass scale of 1 TeV. Lowering  $E_e$  to 70 GeV, the reach of the seesaw mass scale is also lowered, but can still have more than a thousand events to be studied for seesaw mass scalae as high as 700 GeV. For the ST-III model, more than a hundred events will be accumulated when  $m_N < 700$  and  $m_N < 400$  GeV for the case of  $E_e = 140$  and  $E_e = 70$  GeV, respectively. With e-p collision mode we can use the process  $e^- p \rightarrow E_{1,2,3}^- j + X \rightarrow \tau^- Z j + X$  with  $Z$  into hadron jets to study the heavy charged leptons in the ST-III model, whose cross section can reach a few  $\text{fb}$  when the heavy charged lepton mass  $m_E < 600$  and 400 GeV for  $E_e = 140$  and 70 GeV, respectively.

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